Article

Pregnant Zero and Universal Paradox

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ABSTRACT

To illustrate the nature of paradox and sexual division we shall first make a short digression into numbers and logical systems including how to bootstrap from nothing to an abnormal universe. Paradox, in terms of logically confounding, has a wider application. Another face of sexual paradox may lie in our incapacity to completely tie down descriptions of reality into fully-definable closed systems. Thus, logical systems are rather like open thermodynamic systems, which exchange information across their boundaries, and do not necessarily tend to a closed equilibrium. There is also a deep identity between logic and set theory, because the set operations are symbolic logic applied to the elements. Finally, mathematics, like quantum reality, contains two currents, typified by the discrete operations of algebra and combinatorics and the continuous properties of the functions and limit operations of calculus and topology.

Key Words: pregnant, zero, universal paradox, logic, set theory, algebra, topology, calculus.

As a koan for sexual paradox we can consider the double contradiction represented by a single sheet of paper with contradictory signs on each face:

The statement on the other side is false.
The statement on the other side is true.

If we accept either in its entirety, we are in a double-bind, for each leads us into a state of global contradiction, when the other is taken into account. In fact paradox is itself sexual between truth and falsehood for it is a situation where the truth of the statement implies its falsehood and vice versa. In every other situation truth and falsehood remain segregated but in paradox they are in logical coitus.

"I Wonder if They Had Trouble Inventing Zero?"

To illustrate the nature of paradox and sexual division we shall make a short digression into numbers and logical systems. This is taken with due respect to the vastly greater complexity of the real world of living beings, consciousness and the enigmas of quantum reality simply to go to the heart of logical paradox and its implicitly sexual nature permeating existence.

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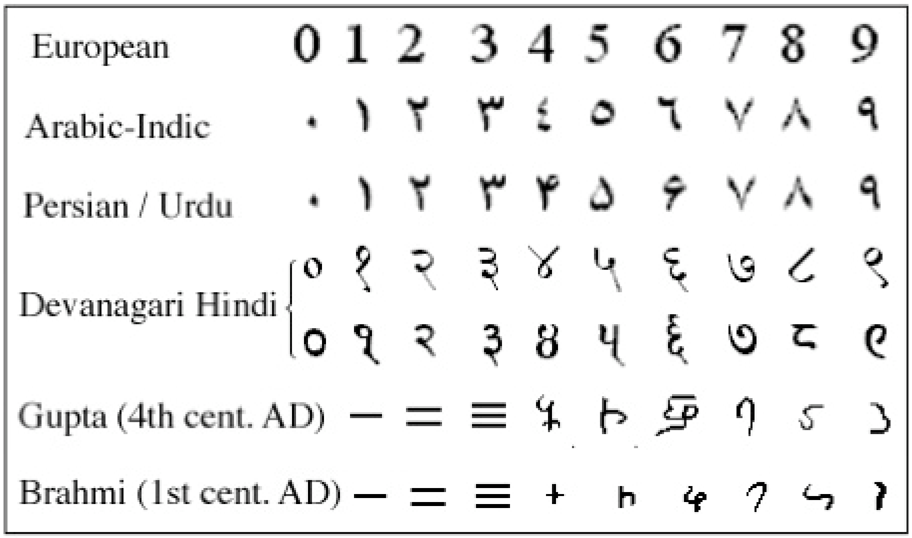


Arabic-Indic numerals well-indicate the nature of such primal division. The linear 1 is an inheritance from a simple mark, a line, digit or finger, indicating the presence of an object, or event, thus making a primal distinction - existence. By contrast, zero is much more difficult to arrive at, since it does not count for anything at all and at face value there is nothing to be done. Zero thus only came to be recognized indirectly, through a long, tortuous route, which first led to positional notation for numbers and then to the need for a mark for a space For example in counting a hundred and one objects by writing 101, the zero (*sunya*) is needed to indicate there are no tens. This led to the use of a simple dot (*bindu*) to indicate a space and eventually the small circle we now associate with zero. In the process even the stars in the sky came to be referred to in Indian literature as *sunya bindu* or 'void dots'.

Zero and with it positional notation was transmitted in the Middle ages from India, through the Arabs, to Europe. In the process bindu became Arabic sifr which again means a space. This has evolved into our term cipher, which has come to mean both zero and a digit, with undertones of worthlessness - 'a mere cipher' and more sinisterly, covert encryption - to 'decipher' a code is to expose it - illustrating the dark concealed complexity within zero in the minds of men seeking a universe of perfect order.

Zero, and with it the positional number system, was championed by Fibonacci, whose name, 'Bigollo', can mean 'good-for-nothing' as well as 'traveller', both of which are apt here. Fibonacci is among other things famous for the sequence of magic numbers we shall see play a role in chaos (p 504). It is thus to Fibonnaci all merchants owe the Arabic-Indic positional notation which has made numerical calculation so natural and made both science and the precipitous rise and fall of stock markets in their triple witching hour possible.

Positional notation was actually a much earlier invention, harking back to Sumeria and Babylon, but here it was embraced in base sixty, much too large for easy manipulation of less than titanic quantities, and so succeeding cultures lapsed back into the positionless numbers exemplified by Roman numerals such as MCMXLIV which are unnaturally clumsy to calculate with. Our current base 10 number system, which transparently corresponds to our two sets of five appendages, shared between vertebrates and echinoderms, still declares its biological origin in the name 'digits' given to whole numbers. In a sense the numbers other than zero are simply 'giving the fingers'.

In a final nemesis of the historical complexity of number bases, the modern computing paradigm has converged back to the dyadic sexual division itself in the two primal numerals 0 and 1 in using base 2 arithmetic because this reflects the two states of existence of a single discrete 'charge' in memory - a binary distinction, now symbolized by an almost 'penile' 1 complemented by a distinctly 'vaginal' 0.

The 'it from bit' concept of quantum cosmology attempts to similarly describe all quantum phenomena in terms of their information content in terms of bits signified again by the binary 'digits' 0 and 1, sometimes with the three-valued logic of 0, 1 and 'uncertain'.

Despite their fertile and convoluted history, the numerals in their binary form return to a primary sexual division between a pregnant feminine 0 representing absence, enclosure and completion and a discriminating masculine 1, representing existence, distinction and separation. Enclosure and completion lead naturally to wave-like nature while distinction and separation to particle nature. Notice also that the absence implied by the zero is also both a unity undivided by distinction and a totipotentiality out of which all distinction can come, just as the primal void or chaos of *tohu vohu*.

These two processes are fundamental to all differentiation of the existential condition, in both the physical and experiential realms. For anything to exist, it must be differentiated from that which it is not. The primary distinction is between existence manifest as 1 and non-existence represented by the positional void of 0. The totality is thus in its essence a unity which is also a duality. This is a primal form of sexual paradox at the origin of the cosmos. Any attempt to draw a distinction by placing a boundary also results in enclosure, so the zero also contains within it the germ of the 1 just as the 1 also implies the unity of existence and thus contains the germ of 0.

Sexual paradox implies a primal cosmology, in which all phenomena present as complements, through which the totality is manifest. These result in contradiction or degeneracy if either of the primal complements is invoked to the exclusion of the other. The cosmos described as divine order in the human mind leaves the totality lacking the chaotic abyss underlying zero which is pregnant because it is the formless progenitor of new form, just as we shall see new order frequently bifurcates out of chaos. This means that the feminine aspect of the zero harbours undelimited potential as well as the instability and fluctuation abhorred in descriptions of a cosmology emerging from primal chaos.

Bootstrapping from Nothing to an Abnormal Universe

We can see this potentiality of the pregnant 0 in archetypal form in Von Neumann's construction of the natural numbers from the empty set $\varnothing$ - the set consisting of no elements or members, and thus corresponds to the cardinal number 0. We can then recursively define $\{\varnothing\}$ the set whose only element consists of the empty set, which corresponds to cardinal number 1 and then the set consisting of $\{\varnothing,\{\varnothing\}\}$ corresponding to 2 and so on, thus defining all the natural numbers from the 'pregnant' empty set - form out of void.

The very notion of a set of elements attempts to give us a foundation level of distinction and membership, $x \in X$, defining x to be an element of the set X. Notice also that attempts to define a universal set run into a paradox due to Bertrand Russell which is also in its essence a sexual paradox of division associated with the concept of a universal totality in which elementary membership is possible.

Notice that our construction of the numbers from the empty, or null set assumed that a set such as $\varnothing$ could itself be an element of another set $\{\varnothing\}$. We however immediately run into trouble when we consider the universal set U consisting of all possible sets. Let us define a set to be normal if it does NOT contain itself as an element - i.e. $X \notin X$... Now, if we consider N the set of all normal sets, we find we are in a double bind. Is N normal or not? If it is normal then it is not a member of itself and hence not a member of the set of all normal sets, hence it is abnormal. But if it is abnormal then it is a member of itself and hence a member of the set of all normal sets and hence normal. Contradiction.

This universal paradox is both sexual in the sense that it arises from a division into two conditions, normal and abnormal, and is fundamental to the notion of universality even in the restricted domain of 'normality'. We can't eliminate the dilemma of Russell's paradox no matter how counter-intuitive it may seem. Sexual paradox is thus primal to the universal order. Notice that it also fulfils an important part of the central idea of paradox, as something which defies common sense and is yet true.

Propositions which Can't Make up their Minds

However paradox, in terms of logically confounding, has a wider application. Another face of sexual paradox may lie in our incapacity to completely tie down descriptions of reality into fully-definable closed systems. Traditionally a contradiction is a statement whose truth implies its falsehood and vice versa. This is rather like a logical version of the entangled mixing we shall later associate with chaotic processes, in the sense that true and false have become inextricably entwined. However, the other face of this is an undecidable proposition - one to which neither true nor false can be definitively be assigned from the axioms of the system.

Kurt Godel has become famous or notorious, depending on your point of view, for proving his 'incompleteness theorem': that any logical system containing finite arithmetic possesses within it formally undecidable propositions. This means also that the propositions definable within real world systems can never be exhaustively proven, because some of the statements we can validly make can never be verified using the system's postulates.

Peano arithmetic, has five simple axioms defining natural numbers, starting from 0 using successors: $1=S(0)$, $2=SS(0)$, etc. The axioms are as follows for any a and b:

$$(1)\ Sa \neq 0,\ (2)\ a+0=a,\ (3)\ a+Sb=S(a+b),\ (4)\ a.0=0,\ (5)\ a.Sb=a.b+a$$

Godel's trick was to also use the natural numbers to encode logical propositions. We can do this encoding by assigning unique numbers for free variables such as a and b, for arithmetic operators such as '+', '.', '=' and so on, and also for the logical statements 'for all', 'there exists', 'such that', 'not', 'and', 'or', etc. All statements provable from the axioms can be reached by syntactic substitutions which can also be encoded using the number code.

Around twelve simple rules of substitution, such as:

'*for all a this does not happen*' = '*there does not exist a for which this happens*'

can generate every syntactically correct proposition in Peano arithmetic, and each proposition can thus be represented by a unique integer, called its Godel number. Given the five axioms and some twelve grammatical rules for constructing statements, this 'numerical logic' lets us also write down propositions which are about propositions. Not all Godel numbers correspond to syntactic statements which can be proved from the axioms however. Each that can be proved we can call a 'theorem of numerical logic'. In particular, we can write down self-referential propositions, ones which include their own Godel number.

The key Gödel number that is undecidable corresponds to a self-referential paradox:

"This statement is not a theorem of numerical logic."

Is the above statement true or false? If it is false, then it IS a theorem of numerical logic, so we have a valid theorem proved from the axioms, but it is false, so the whole system falls apart by inconsistency. Hence it must be true. But if it is true, then, by its own statement, it is not a theorem of numerical logic. This means that the statement is true, but it is not a provable proposition from the axioms. The system is thus incomplete, and the truth of the proposition is undecidable within the system.

Undecidable propositions can come in diverse and varied forms, but a classic one is: "Will a computational process eventually complete?" - the 'halting problem' of Alan Turing, who broke the 'enigma' code. It is illustrated (p 382) in cellular automata. Even for these apparently simple systems, we cannot determine in advance whether they will terminate or reproduce forever. Such issues extend to cellular automata simulating the prisoners' dilemma game (p 21), which can also be undecidable (Grim R270).

Can the Universe Be Told?

The second law of thermodynamics says that any closed system will tend to equilibrium where entropy (disorder) is at a maximum and the rate of entropy production has fallen to a minimum of zero. Ilya Prigogine (R524) and others have noted that open thermodynamic systems however do not have to tend to equilibrium but may go into complex far-from equilibrium states where entropy production is minimized . The living biosphere is an example in which increasing complexity and decreasing entropy result from the open thermodynamic boundary of the biosphere fixing incident solar energy by photosynthesis.

Logical systems are thus rather like open thermodynamic systems, which exchange information across their boundaries, and do not necessarily tend to a closed equilibrium. Because Godel incompleteness also applies to any logical system containing finite arithmetic it applies to virtually all non-trivial domains of existence including the real world around us with its manifold complexities of quantum reality, life and subjective experience, which transcend formal logic. After spending years seeking a unified theory of cosmology, Stephen Hawking has recently begun to lament that Godel's theorem may make it impossible to form a grand unified 'theory of everything', or TOE, for the universe.

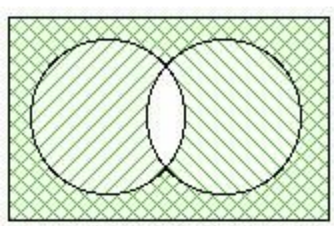
The Tao te Ching expresses this succinctly as "The way that can be told is not the countless way". Transcendence over closed logic is common to many religious and mystical traditions. We are here running up against a form of paradox involving order, chaos, complexity and our incapacity to define a closed boundary between a given realm and its complement. It is pertinent here to consider our linguistic terms for the universal condition and ponder the role mankind's will-to-order plays in determining our universal concepts and the toll it has on the darker concealed aspect the pregnant zero contains.

The founding historical concept in both the religious and natural realm is 'cosmos' from the Greek kosmos - order. Cosmos here, despite laying claim to be the entire universe is implicitly contrasted with primal chaos as the condition of divine or perfect order. Norman Cohn's work "Cosmos Chaos" (R98) well describes the counterpoint of these two ideas in religious history, from the slaying of Tiamat - the primal mother waters of chaos by Marduk the divine agent of civic order, through the Biblical and Apocalyptic traditions to the founding of Christianity. 'Universe', our more physically accepted scientific term, again highlights an evolution to unity through order in the Latin *universus* - 'turned into one' - *uni*- 'one' *versere* 'turned'. Such an ordered unity is a little like the 'false vacuum' we will investigate at the inflationary cosmic origin. 'Turned into one' is an imposed order violating the ground swell of chaos and unstable to it into which imposed order shall again dissolve in the final unravelling (p 352).

The Symmetry-Breaking Logic of Topology

There is a deep identity between logic and set theory, because the set operations are symbolic logic applied to the elements. Union, intersection and complement are simply logical OR, AND and NOT:

$$A \cup B = \{x \in X, (x \in A) \vee (x \in B)\}$$
$$A \cap B = \{x \in X, (x \in A) \wedge (x \in B)\}$$
$$X \backslash A = \bar{A} = \{x \in X, \neg(x \in A)\}$$



$X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$

Moreover these laws are symmetric to union and intersection, to form a Boolean algebra.

For example we have the distributive laws

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

and De Morgan's laws of complements

$$X \backslash (A \cup B) = (X \backslash A) \cap (X \backslash B)$$
$$X \backslash (A \cap B) = (X \backslash A) \cup (X \backslash B)$$

Topology breaks this symmetry, by defining open sets such as

$$(0, 1) = \{x \in R, 0 < x < 1\}$$

to exclude their boundaries and consist only of interior points and closed sets such as

$$(0, 1) = \{x \in R, 0 < x < 1\}$$

to include all their boundary points. Complements of open sets are closed and vice versa, for example

$$R \backslash (0, 1) = (-\infty, 0] \cup [1, \infty),$$

which is closed, since it contains its only two boundary points, 0 and 1. However most sets such as

$$(0, 1] \cup [2, 3)$$

are neither open nor closed.

We now find open sets remain open under infinite unions but only finite intersections:

For example:

$$\bigcup_{n \in N} \left(\frac{1}{n}, 1 - \frac{1}{n} \right) = (0, 1)$$

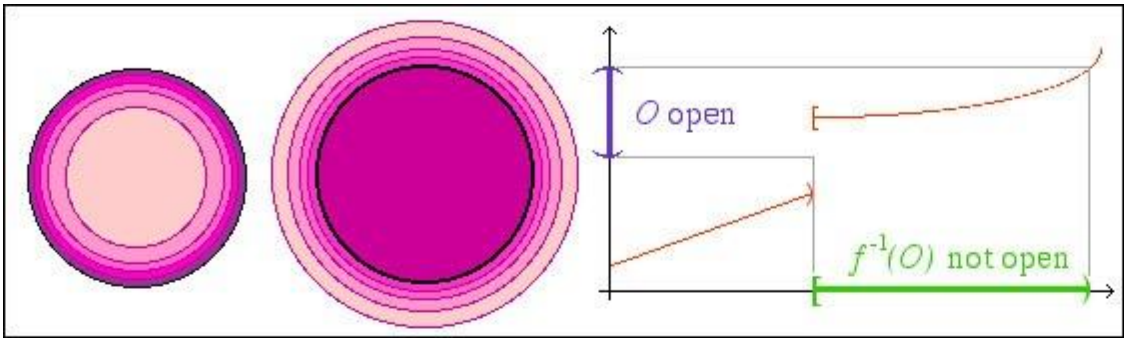
but

$$\bigcap_{n \in N} \left(\frac{-1}{n}, 1 + \frac{1}{n} \right) = [0, 1].$$

This symmetry-breaking becomes the axioms for a topological space, upon which all continuity depends. Arbitrary unions of open sets are open but only finite intersections are:

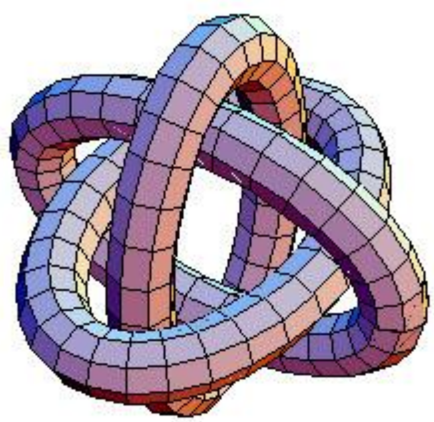
$$(1) \quad \{O_i \text{ open}, i \in I\} \Rightarrow \bigcup_{i \in I} O_i \text{ open}$$

$$(2) \quad \{O_i\,(\text{open}, i, 1\ldots n)\} \Rightarrow \bigcap_{1\ldots n} O_i \ \ \text{open}.$$



(a)Symmetry-breaking in topology. The union of open discs of radius $r < 1\text{-}1/n$ is the open disc of radius $r < 1$, but the intersection of open discs of radius $r < 1+1/n$ is the closed disc of radius $r <= 1$. (b) A discontinuity in a function is detected because the inverse image of open *O* is not open. Continuous functions, including waves, are the basis of differentiation and integration in calculus; representing the function's instantaneous rate of change, or slope; and cumulative area enclosed.

Continuity can be defined in terms of open sets. A function $f: X \to Y$ is continuous if and only if, for every open set *O* in *Y*, the inverse image $f^{-1}(O) = \{x \in X, f(x) \in O\}$ is open .



Borromean rings as 2-D manifolds illustrate how continuity becomes the basis of defining knotted topological spaces. Such ideas may be implicit in the transformations required to integrate gravity with the other forces (p 314) and may use a complementary type of brain activity to formal linguistic processes of symbolic manipulation (p 367).

Mathematical Foundations of Complementarity

Mathematics, like quantum reality, contains two currents, typified by the discrete operations of algebra and combinatorics and the continuous properties of the functions and limit operations of calculus and topology. Indeed as noted (p 350) it has been suggested that these complementary aspects of mathematics may be lateralized, with the knotty topological properties of continuity being a right brain spatial activity, and the symbolic manipulations of algebra being a left-brain activity like language. Although mathematicians will hasten to obscure this distinction by pointing out the essential unity of mathematics and the fact that algebra and calculus deal with both discrete and continuous systems, the distinction is intriguingly reflected in the mathematics used to discover the fundamental principles of quantum theory.

Werner Heisenberg (R760).

Heisenberg was the first person to define the concept of quantum uncertainty, or indetermincay, as the term also means in German.

Heisenberg's research concentrated on momentum and angular momentum. It is well known both rotations in 3-D space and matrices in general do not commute. $AB \neq BA$, because matrix multiplication multiplies the rows of the first matrix by the columns of the second:

$$AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix},$$

but

$$BA = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{bmatrix}.$$

Hence

$$AB - BA \neq 0.$$

More generally, if $C = AB$,

$$C_{rc} = \sum_s A_{rs} B_{sc}.$$

In quantum mechanical notation, we have

$$\langle r|A|c\rangle = A_{rc} \text{ so } \sum_s \langle r|A|s\rangle\langle s|B|c\rangle = \sum_s A_{rs} B_{sc} = [AB]_{rc} = \langle r|AB|c\rangle,$$

showing that

$$|s\rangle\langle s| = 1,$$

all states leading to completeness with unit probability.

Erwin Schrodinger (R760)

Schrodinger's wave equation and Heisenberg's matrix mechanics highlight a deeper complementarity in mathematics between the discrete operations of algebra and the continuous properties of calculus. When Heisenberg was trying to solve his matrix equations, the mathematician David Hilbert suggested to look at the differential equations instead. But it fell to Schrodinger, who took his mistress up into the Alps and discovered his wave equation on a romantic tryst. It was only when Hilbert and others examined the two theories closely that it was discovered they were identical, but complementary, descriptions.

Schrodinger derived his time-independent wave equation as follows. The Hamiltonian dynamical operator representing the total kinetic and potential energy $H = K + V$, of the system, in terms of how the wave varies with time and space:

$$H(\bar{r}, t)\Psi(\bar{r}, t) = \left(\frac{-h^2}{8\pi^2 m}\nabla^2 + V(\bar{r})\right)\Psi(\bar{r}, t) = \frac{ih}{2\pi}\frac{\partial}{\partial t}\Psi(\bar{r}, t),$$

Where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

This is a non-relativistic equation expressed in terms of the first time derivative. If we now assume the wave function consists of separate space and time terms

$$\Psi(\bar{r}, t) = \psi(\bar{r})\phi(t),$$

and seek time independence of the wave function at constant energy *E*, we get

$$H(\bar{r})\psi(\bar{r}) = \left(\frac{-h^2}{8\pi^2 m}\nabla^2 + V(\bar{r})\right)\psi(\bar{r}) = E\psi(\bar{r}),$$

or

$$H\psi = E\psi.$$

Interpreted in terms of matrix mechanics, the Schrodinger wave equation becomes a sum of basis vectors

$$\psi = \sum_n c_n|n\rangle$$

representing each of the wave states. The algebraic version of the equation ,

$$H\psi = E\psi,$$

becomes

$$H\sum_n c_n|n\rangle = \sum_n c_n H|n\rangle = E\sum_n c_n|n\rangle.$$

Solving in terms of a transformation to a new state, we have

$$\langle m|H\sum_n c_n|n\rangle = \sum_n c_n\langle m|H|n\rangle = E\sum_n c_n\langle m|n\rangle,$$

where

$$\langle m|n\rangle = \begin{cases} 0, m \neq n \\ 1, m = n \end{cases}.$$

Hence

$$E\sum_n c_n \langle m|n\rangle = Ec_m$$

and so

$$\sum_n H_{mn} c_n = Ec_m.$$

Thus

$$H_{nm} = 0,\ m \neq n$$

and

$$H_{mm} c_m = Ec_m.$$

This the famous eigenvalue (own-value) problem, whose stable standing wave solutions are the *s*, *p*, *d* and *f* orbitals of an atom (p 318).

Heisenberg's problem of uncertainty expressed in non-commuting operators such as position x and momentum gives us back the uncertainty relation when we reinterpret momentum in terms of the wave function as a differential operator

$$p_x = \frac{h}{2\pi i}\frac{d}{dx},$$

we have

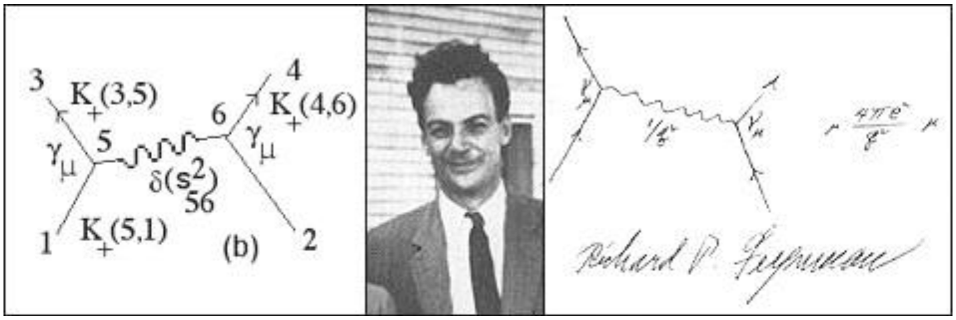
$$(xp_x - p_x x)\Psi = x\frac{h}{2\pi i}\frac{d\Psi}{dx} - \frac{h}{2\pi i}\frac{d}{dx}(x\Psi) = x\frac{h}{2\pi i}\frac{d\Psi}{dx} - \frac{h}{2\pi i}\left(\Psi\frac{dx}{dx} + x\frac{d\Psi}{dx}\right) = -\frac{h}{2\pi i}\Psi = \frac{i}{h2\pi}\Psi.$$

Hence

$$[x, p_x] = (xp_x - p_x x) = i\frac{h}{2\pi},$$

another view of the uncertainty relation

$$\Delta x \Delta p_x \sim \frac{h}{2\pi}.$$



Feynman diagram for first order photon exchange in electron-electron repulsion. Richard Feynman with his own diagram (R760).

The underlying wave-particle complementarity in Feynman's approach to quantum field theory, despite its apparent explanation of the electromagnetic field in terms of particle interaction is succinctly demonstrated in the first-order diagram from electron-electron scattering (electromagnetic charge repulsion) through exchange of virtual photons provided by uncertainty. The propagator for the diagram is:

$$K_{3,4,1,2} = -e^2 \iint K_a(3,5) K_b(4,6) \gamma_{a\mu} \gamma_{b\mu} \delta(s^2{}_{56}) K_a(5,1) K_b(6,2)(d\tau_5) d\tau_6$$

where γ are the variants of the Pauli spin matrices, the Dirac δ function represents the discrete interaction of the virtual photon over the space-time interval, $s^2{}_{56}$ and $K$are the propagators for electrons a and b to be carried by Huygen's wave-front principle according to the wave summations

$$K(p,q) = \sum_{posE_n} \varphi_n(2)\overline{\varphi_n}(1) e^{-iE_n(t_2 - t_1)}$$

for$t_2 > t_1$ representing positive energy 'retarded' solutions travelling in the usual direction in time and

$$K(p,q) = -\sum_{negE_n} \varphi_n(2)\overline{\varphi_n}(1) e^{-iE_n(t_2 - t_1)}$$

for the corresponding negative energy solutions in the reversed 'advanced' time direction, where E_n and φ_n are the energy eigenvalues and eigenfunctions for the wave equation.$t_2 < t_1$

This both explains how the relativistic solution gives rise to both time backward negative energy solutions and time forward positive energy ones, which make particle-anti-particle creation and annihilation events critical to the sequence of Feynman diagrams possible, and also shows clearly in the complex exponentials the sinusoidal wave transmission hidden in the particle diagrams of the quantum field approach.