Article

Relativistic Energy-Momentum & Relativistic Quantum Mechanics

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ABSTRACT

The relativistic energy-momentum relationship is far more subtle than it seems to be at a first sight. As concerns the relativistic quantum mechanics, its underlying equations were deduced from, or in relation with the relativistic energy-momentum relationship by means of the principle of correspondence. Without a clear physical role and meaning associated, the matrices α, β of the Dirac equation seemed to confirm that the principle of the physical determination of equations would not be proper to the new quantum mechanics. Therefore, we have to search for genuine physical information in the terms of the underlying equations of the relativistic quantum mechanics. This information concerns a level of structure of matter "even below that on which nuclear transformations take place."

Key Words: relativistic, energy-momentum, quantum mechanics, Dirac equation.

25. ABSOLUTE RELATIVISTIC ENERGY

In special relativity theory, the energy of a particle (*E*) is relative quantity and mentioned with the linear momentum (***p***) of the particle. Both of them are the components of the four-vector $p^\mu=(\boldsymbol{p},E/c)$ called four-linear momentum. Both *E* and ***p*** depend (by their defining relationships $E=\beta m_0c^2$ and $p=\beta m_o v$) on the speed v of the particle with respect to an inertial observer. They change under the Lorentz transformation, but $p^\mu p_\mu$ is invariant, equal with the rest energy of the particle by the relativistic energy-momentum relationship

$$E^2/c^2-p^2=m_o{}^2c^2. \qquad (28)$$

The lack of the 'abstract' coordinate systems at absolute rest determined Einstein to consider the concept of proper frame. That his decision was a wrong one, is evident. By definition, the rest energy of a particle with respect to the proper frame is m_0c^2. This is so because the particle is (by definition) at rest with respect to this frame. But as the reference frame at absolute rest was excuded from special relativity theory, the reference frame in which a particle is at rest is an inertial one. So the particle moves actually through space at the speed of this reference frame. If the 'blind' inertial observer cannot determine experimentally the state of motion of his reference frame, and so the energy and linear momentum through space of a particle at rest in this frame, is just the theory's fault. It is the reason for which we opted for professionals (Sec. 6) in special relativity theory.

As the energy of a particle is an objective quantity, its definition with respect to inertial reference frames was misleading. The only energies that an inertial observer could get practically, in consequence, were restrained to those he defined with respect to his reference frame that is a part

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of the particles' kinetic energy. Einstein's definition of the rest energy with respect to the proper frame was merely wrong.

Validating both the 'abstract' coordinate systems at absolute rest and the absolute speed in special relativity theory, m_0c^2 appears naturally as the energy of a particle at absolute rest, while βm_0c^2, with v standing now for the absolute speed, as the energy of a particle defined as absolute quantity (Sec. 6 (Sect. 1)). As any inertial observer can determine experimentally absolute speeds, he can also determine the absolute energies of the particles. Consequently, he can develop adequate tools to exploit this energy. Einstein's arbitrary removal of the concepts of absolute rest and absolute speed just forbidden for a century the exploitation of the absolute energy.

26. CONCERNING THE RELATIVISTIC ENERGY-MOMENTUM RELATIONSHIP

The relativistic energy-momentum relationship is far more subtle than it seems to be at a first sight. With the usual meaning of relative quantities of its terms, Eq. (28) was written exclusively with respect to the reference frames of the inertial observers Our identification of the terms in Eq. (28) also as absolute quantities [45-47] define also Eq. (28) with respect to coordinate systems at absolute rest. There becomes evident that β in *E* only coincides with the β in the Lorentz transformation, as long as an observer moving with absolute speed v also sees βm_0c^2 as the energy of a particle at absolute rest. With the meaning of absolute quantities of its terms, the relativistic energy-momentum relationship validates the principle of the physical determination of equations in the relativistic quantum mechanics (Sec. 27). This means that genuine subquantum information, complementary to that obtained by colliding high energy particles, is to be disclosed from the terms of the underlying equations of this theory. We do it in the next part of this book. Most important is the information concerning the subquantum energy, on which will be founded radically new technologies like those pointed out in Sec. 41. Unfortunately, the way to disclose and apply such information is firmly forbidden by the perennial wrong physics policy based on disregarding the concepts of absolute rest and absolute speed.

Finally, we point out a peculiarity of Eq. (28). Observe that multiplyed by $\gamma^2=1/(1-v_1^2/c^2)$, where v_1 stands for the speed of a particle at rest in the one observer's reference frame, and putting $m=\gamma m_0$, $E=\beta mc^2$ and $p=\beta mv$, Eq. (28) is written with respect to the reference frame of the inertial observer. As concerns this multiplication, it is mathematically doubtless, but physically incomprehensible now. It is under this form that the relativistic energy-momentum relationship gives evidence for the subquantum nature of the relativistic mass in the manner in which it does it like Eq. (28) for the rest mass in Sec. 27.

27. VALIDATING THE PRINCIPLE OF THE PHYSICAL DETERMINATION OF EQUATIONS IN RELATIVISTIC QUANTUM THEORY. TOWARD GENUINE INFORMATION ON THE STRUCTURE OF SOME 'ELEMENTARY' PARTICLES

The relativistic theories were built on Einstein's special relativity theory, and 'developed' at the time the principle of the physical determination of equations was not validated in the special relativity theory. As concerns the relativistic quantum mechanics, its underlying equations were deduced from, or in relation with the relativistic energy-momentum relationship by means of the principle of correspondence. Without a clear physical role and meaning associated, the matrices α,β of the Dirac equation seemed to confirm that the principle of the physical determination of equations would not be proper to the new quantum mechanics. So that, all the physical information that Dirac obtained for a free particles was by resolving the equation carrying his name, and concerned its quantum behavior. The Dirac particles remained further 'elementary' particles for him.

A major step further was done, in principle, by der Waerden. His revealed idea (revealed because he never became aware of its physical significance and consequences) of writing the term $\mathbf{p}^2$ in the relativistic energy-momentum relationship like $(\boldsymbol{\sigma}\cdot\mathbf{p})(\boldsymbol{\sigma}\cdot\mathbf{p})$, where $\boldsymbol{\sigma}=(\sigma_x,\sigma_y,\sigma_z)$ are the Pauli 2x2 matrices associated to the spin operator $(\hbar/2)\boldsymbol{\sigma}$ and $\hbar$ is the reduced Planck constant, rendered this relationship more fit to the internal structure of the Dirac particles. His derivation of the spinorial transcription of the Dirac equation confirms this assertion. Unfortunately, by virtue of the mathematical equivalence of all the transcriptions of the Dirac equation, der Waerden's derivation of the spinorial transcription rested as good as any other.

Since we validated the principle of the physical determination of equations in the special relativity theory, and the energy-momentum relationship is also basic for the relativistic quantum mechanics, this principle is (by Sec. 26) valid in the relativistic quantum mechanics, too. Therefore, we have to search for genuine physical information in the terms of the underlying equations of the relativistic quantum mechanics. This information concerns a level of structure of matter "even below that on which nuclear transformations take place" (Bohm, [48]).

Investigating the terms of the spinorial transcription of the Dirac equation as it was deduced by der Waerden, and the wavefunctions corresponding to opposite eigenvalues of both the helicity and velocity operators in which the Dirac wavefunctions are equally splitting, we obtain information condensed in one model of Dirac particle consisting of two coupled systems of subquantum particles spinning tangentially in opposite directions. Two systems of subquantum particles spinning in opposite directions we also identify inside photons, and suggest their existence inside spin-0 mesons. This model of 'elementary' particle has nothing in common with the mathematical standard model of 'elementary' particle, and is important by that it defines the relativistic mass as the coupling constant of the systems of subquantum particles, and allow developing radically new technologies by altering this constant.

28. INFORMATION PROVIDED BY THE SPINORIAL TRANSCRIPTION OF THE DIRAC EQUATION

Consider der Waerden's derivation of the spinorial transcription of the Dirac equation [49] from the relativistic energy-momentum relationship written in the form

$$(E/c+\boldsymbol{\sigma}\cdot\mathbf{p})(E/c-\boldsymbol{\sigma}\cdot\mathbf{p})=m_oc^2 \qquad (29)$$

where c is light speed, E is the energy, $\mathbf{p}$ the linear momentum and m_0 the rest mass of a free particle. By applying the principle of correspondence, der Waerden passed from the physical quantities E and $\mathbf{p}$ in Eq. (29) to the suitable quantum operators $E=i\hbar\partial_0$, $\mathbf{p}=-i\hbar\boldsymbol{\partial}$, where $\partial_0=\partial/\partial(ct)$ and $\boldsymbol{\partial}=(\partial/\partial x,\partial/\partial y,\partial/\partial z)$. He applied the resulting equation to the two-component spinor ξ, and put

$$(i\hbar/m_oc)(\partial_o-\boldsymbol{\sigma}\cdot\boldsymbol{\partial})\xi=\eta, \qquad (30)$$

where ξ and η correspond to the same spin polarity. He thus obtained from Eq. (29) the set of equations

$$i\hbar(\partial_o+\boldsymbol{\sigma}\cdot\boldsymbol{\partial})\xi=m_oc\eta,\quad i\hbar(\partial_o-\boldsymbol{\sigma}\cdot\boldsymbol{\partial})\eta=m_oc\xi \qquad (31)$$

which constitute the spinorial transcription of the Dirac equation, a step toward the covariant Dirac equation.

According to the validity of the classical principle of physical determination of equations in Einstein's special relativity theory, and despite the mathematical equivalence of all the transcriptions, we search for physical information on the internal structure of the Dirac particles in the terms of der Waerden's derivation of Eqs. (30). To this end, we first consider the equation

$$i\hbar\partial_o\psi_1=(E/c)\,\psi_1+(K/c)\psi_2, \qquad (32)$$

describing a weak coupling in the quantum mechanical formalism, where ψ_1 and E are, respectively, the eigenfunction and the suitable eigenvalue of the Hamiltonian equation while K and ψ_2 are, respectively, the coupling constant and the contribution of that coupling to the eigenstate ψ_1. By comparing each of Eqs. (31) with Eq. (32), we see that 1) Applied to ξ and η, the Hamiltonians $\pm\bar{\sigma}\cdot\mathbf{p}$ describe opposite spin-momentum couplings within a free particle of well-defined direction of the linear momentum, which means that two internal entities spin in opposite directions, 2) $m_0c^2$is coupling the two entities, and 3) A leakage of subquantum constituents between the two entities is assumed. According to Bohm, m_0c^2 is the energy of a particle "having no visible motion as a whole" [48], and originates in "to and fro reflecting movements" [50]. Our result recovers m_0c^2 as the energy of a particle "having no visible motion as a whole", i.e., a particle at absolute rest, but discloses that m_0c^2 actually originates in the coupling of the particle's constituents, being a subquantum coupling energy. The particle rest mass m_0 is the true coupling constant. Our result does not presume the existence of a physical coupling between the particle spin and its linear momentum, but the effect that a change in the particle speed has upon its internal coupling. The two entities are systems of subquantum particles. Our result is refined to a semi-classical model of Dirac particle at absolute rest by regaining the maximal acceleration $\mathbf{a}_{\mathrm{M}}=2m_oc^3/\hbar$ as a subquantum quantity (Sec. 29), and constructing the spinning frequency operator for Dirac particles (Sec. 33).

29. MAXIMAL ACCELERATION AS SUBQUANTUM QUANTITY. SEMI-CLASSICAL MODEL OF DIRAC PARTICLE

Consider the maximal acceleration

$$\mathbf{a}_{\mathrm{M}}=2m_oc^3/\hbar.$$

It was derived by embedding an eight-dimensional metric in phase space, as well as by means of the Heisenberg uncertainty relations [51-54]. By its dependence on the reduced Compton wavelength $\lambda\!\!\!^{-}_C=\hbar/m_oc$, and because $\lambda\!\!\!^{-}_C$ is the reduced wavelength of the de Broglie wave of a particle at absolute rest (Sec. 30), $\mathbf{a}_{\mathrm{M}}$ belongs to micro-world.

Our derivation of $\mathbf{a}_{\mathrm{M}}$ as the acceleration of a spinning sphere of radius r= $\lambda\!\!\!^{-}_C/2$ and peripheral speed *c* by $a=c^2/(\lambda\!\!\!^{-}_C/2)$ confirms that $\mathbf{a}_{\mathrm{M}}$ is a subquantum quantity. The acceleration $\mathbf{a}_{\mathrm{M}}$ -due to the change in direction of its peripheral speed- belongs to a sphere the diameter of which is half that of a quantum particle. Therefore, our semi-classical model of Dirac particle at absolute rest consists of two coupled spherical systems of subquantum particles of radius $\lambda\!\!\!^{-}_C/2$ that spin tangentially. To assure the stability of the particle, the two systems can spin only in opposite directions. The spinning frequencies of these systems are

$$\omega = \pm2\omega_o = \pm2m_oc^2/\hbar. \qquad (33)$$

The equal writing of the relativistic energy-momentum relationship with respect to inertial observers (Sec. 26) extends the above results to the relativistic mass.

30. COMPTON WAVELENGTH AS WAVELENGTH OF A DE BROGLIE WAVE

Yet it is stated in the literature that under its reduced form $\bar{\lambda}_C$, the Compton wavelength is just an useful physical constant. Yet there is, as we know, no search for its physical content. Here is the reason for which we were interested in this matter.
Consider the relativistically defined de Broglie relations

$$E = \hbar\omega,\ p = \hbar/\bar{\lambda}, \tag{34}$$

which associate a wave of frequency ⍰ and reduced wavelength $\bar{\lambda}$ to any free particle of energy E and linear momentum **p**. When written for a rest particle of mass m_o, Eqs. (34) reduce to

$$\omega_o = c/\bar{\lambda}_C. \tag{35}$$

Eq. (35) shows that $\bar{\lambda}_C$ is the reduced wavelength of the physical de Broglie wave associated to a rest particle. With this meaning, $\bar{\lambda}_C$ associates by the right hand side of the second of Eqs. (34), and against the linear momentum p=0, the internal momentum (known as the Schrodinger microscopic momentum [55])

$$p_o = m_o c \tag{36}$$

to a particle at absolute rest. With these meanings, both $\bar{\lambda}_C$ and p_o are essential to obtain information on the internal structure of some quantum particles.