Article

Subquantum Dynamics & Wavefunctions

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ABSTRACT

The undulatory phenomenon that de Broglie associated to the quantum particles seems basic for their mathematical description by wavefunctions. The Dirac wavefunctions ψ contains in their structural elements information on the constituents of the Dirac particles responsible for, or at least in interrelation with, the undulatory phenomenon.

Key Words: subquantum, wavefunction, quantum mechanics, Dirac, particle.

31. QUANTUM MECHANICAL RELATIONSHIPS POINTING TO *c* AS SUBQUANTUM QUANTITY

The Dirac spin operator $\mathbf{S}=(\hbar/2)\mathbf{\Sigma}$ gives evidences for *c* as peripheral speed of the spinning systems of subquantum particles by the direct product in the defining relationship [56] $\mathbf{\Sigma}=-(i/2)(\boldsymbol{\alpha}x\boldsymbol{\alpha})=\begin{vmatrix}\sigma & 0\\ 0 & \sigma\end{vmatrix}$, and the commutation relations $[c\alpha_i,\Sigma_i]=0$ ($c\alpha_i$ is the velocity operator, i=1-3): While the defining relationship points to a motion of speed *c* in a plane orthogonal to the spin direction, the commutation relations show, according to the quantum mechanical theory of measurement, that components of the speed non-parallel to one of the spin can not be measured simultaneously with the last. *The validity of our result is supported by that both the Newtonian speed and acceleration as ratios of infinitesimal quantities.*

32. INFORMATION PROVIDED BY THE DIRAC WAVEFUNCTIONS

The undulatory phenomenon that de Broglie associated to the quantum particles seems basic for their mathematical description by wavefunctions, the statistical interpretation of the wavefunctions and experimental performances otherwise impossible to get. Therefore, the Dirac wavefunctions ψ should contain in their structural elements information on the constituents of the Dirac particles responsible for, or at least in interrelation with, the undulatory phenomenon. We just propose searching for such information.

32.1. Splitting the Dirac Wavefunctions in Components of Opposite Helicities

The splitting of the Dirac wavefunctions in wavefunctions of another operator is -by virtue of the principle of the physical determination of equations- essential to obtain information on the structure of the quantum particles. That information is to be identified in their elements.
Focus our attention upon the commutation relation

$$[H_D,h] = 0, \tag{37}$$

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where h is the helicity operator. Eq. (37) assures the existence of a complete set of eigenstates for H_D and h. Although helicity is a good quantum number, Eq. (37) does not specify if the energy eigenstates are helicity eigenstates or linear combinations. To discern between the two possibilities found on equal footing in the literature, we assume that all Ψ are also helicity eigenstates. For a free particle, Ψ is given by

$$\Psi = n \text{ x column } (\psi, k\psi) \text{ x } \exp(ip^{\mu}x^{\mu} / \hbar), \qquad (38)$$

where n is a normalization factor, ψ is as usually a two-component spinor and k is a constant to be determined. The pairs of non-zero values of k that the zero-valued determinants of the systems of second order equations in which Dirac equation splits by inserting Ψ deny such Ψ's.
Consider further the wavefunctions

$$\Psi = \text{column } (\xi, \eta)$$

of the Dirac equation

$$i\hbar\partial_o\Psi = (1/c)H_D\Psi, \qquad (39)$$

where

$$H_D = c\boldsymbol{\alpha}\cdot\mathbf{p} + m_o c^2\beta$$

is the Dirac Hamiltonian, and

$$\boldsymbol{\alpha} = \begin{vmatrix} \sigma & 0 \\ 0 & -\sigma \end{vmatrix}, \ \beta = \begin{vmatrix} 0 & I \\ I & 0 \end{vmatrix},$$

are the Dirac 4x4 matrices, in which Eqs. (31) were joined together.

The eigenfunctions of the equation with proper values associated to Eq. (39) are

$$\psi = \text{column } \{a, b, [(E+cp_3)a + cp_- b]/m_o c^2, [cp_+ a + (E-cp_3)b/m_o c^2]\},$$

where a, b are components of ξ, $p\pm = p_1 \pm i\, p_2$, and normalization factor was ignored.

By a simple calculation, we get -in accordance with (37)

$$\psi = \psi^-_h + \psi^+_h, \qquad (40)$$

where

$$\psi^-_h = (1/2p) \text{ column}\{(p-p_3)a - p_- b, -p_+ a + (p+p_3)b, [(p-p_3)a - p_- b](E-cp)/ m_o c^2, [-p_+ a + (p+p_3)b](E-cp)/m_o c^2\},$$

$$\psi^+_h = (1/2p) \text{ column}\{(p+p_3)a + p_- b, p_+ a + (p-p_3)b, [(p+p_3)a + p_- b](E+cp)/ m_o c^2, [p_+ a + (p-p_3)b](E+cp)/m_o c^2\}$$

are eigenfunctions of h, corresponding, respectively, to negative and positive helicities. The result is found to be independent of representation. As the direction of **p** in space is well-determined, this splitting proves that the Dirac wavefunctions actually provide information on the true existence of

something spinning in opposite directions within a Dirac particle. The result becomes explicit for a particle moving along one of the coordinate axes, particularly along the third axis, when the eigenfunctions ψ^-_h, ψ^+_h are eigenfunctions of Σ_3.
Concluding, it is misleading to associate simultaneously to each of the directions of **p**, and to each state of helicity, positive and negative energy solutions of the Dirac equation. That the physical reality determining the Dirac Hamiltonian and wavefunctions consists in the systems of subquantum particles inhering in a Dirac particle, is best illustrated by Eq. (40): When written for **p**(0,0,p), Eq. (40) turns into a linear combination of eigenfunctions of Σ_3 corresponding to opposite eigenvalues.

32.2. Splitting the Dirac Wavefunctions in Wavefunctions of the Velocity Operator

A simple calculation -in accordance with the commutation relation $[c\boldsymbol{\alpha}\cdot\mathbf{p},\boldsymbol{\Sigma}\cdot\mathbf{p}]=0$ gives

$$\psi=\psi^-_\alpha+\psi^+_\alpha, \tag{41}$$

where

$$\psi^-_\alpha=(1/2p)\ \text{column}\{(p-p_3)a-p_-b,-p_+a+(p+p_3)b,[(p+p_3)a+p_-b](E+cp)/\ m_oc^2,[p_+a+(p-\ p_3)b](E+cp)/m_oc^2\},$$
$$\psi^+_\alpha=(1/2p)\ \text{column}\{(p+p_3)a+p_-b,p_+a+(p-p_3)b,[(p-p_3)a-p_-b](E-cp)/m_oc^2,[-p_+a+(p+p_3)b](E-cp)/m_oc^2\},$$

are eigenfunctions of the operator $c\boldsymbol{\alpha}\cdot\mathbf{p}/p$, which eigenvalues are opposite speeds along the direction of motion. Since the elements $+\boldsymbol{\sigma}$ and $-\boldsymbol{\sigma}$ of $\boldsymbol{\alpha}$ act, respectively, upon □ and □, and

$$(\boldsymbol{\sigma}\cdot\mathbf{p}/p)\xi^+=\xi^+\ [-(\boldsymbol{\sigma}\cdot\mathbf{p}/p)\eta^-=\eta^-],\ (\boldsymbol{\sigma}\cdot\mathbf{p}/p)\xi^-=-\xi^-,\ [-(\boldsymbol{\sigma}\cdot\mathbf{p}/p)\eta^+=-\eta^+],$$

the first two elements of ψ^+_h (ψ^-_h) are identical with the first two elements of ψ^+_α (ψ^-_α), and the last two elements of ψ^+_h (ψ^-_h) are identical with the last two elements of ψ^-_α (ψ^+_α). So that, the splitting of the Dirac eigenstates in helicity eigenstates corresponding to opposite speeds by (7) supports the understanding of *c* as a subquantum peripheral speed of the systems spinning oppositely in the above semi-classical model of Dirac particle.

33. INFORMATION PROVIDED BY THE SPINNING FREQUENCY OPERATOR

The standard way to prove the existence of some physical quantity in quantum mechanics lies in constructing an observable that can, at least in principle, be measured. Accordingly, we define the 'frequency' operator

$$\omega'_i=P_+\omega_iP_++P_-\omega_iP_-.$$

$P_\pm=[1\pm\text{sign}(E)]/2$ are projectors onto positive and negative energy states, ω_i are components of the operator [56] $\omega=-2c\gamma^5p/\hbar$ and γ^5 is the chirality operator. By the relationships

$$P_\pm\omega_iP_\pm=\pm[(\omega_rS_r)\omega_i/E]P_\pm,$$

resulting from a simple but long calculation, we get

$$\omega'_i=(\omega_rS_r)\omega_iH_D\ /E^2. \tag{42}$$

The suitable form of the Dirac Hamiltonian

$$H_D=\mathbf{S}\cdot\boldsymbol{\omega}+m_oc^2\beta$$

in terms of ω'_i is

$$H_D=E^2(\omega'_rS'_r)/p^2c^2,$$

where

$$S'_i=P_+S_iP_++P_-S_iP_-\equiv S_i,$$

for massic particles, and

$$H^0_D=\omega'_r\ S'_r,$$

for massless particles.

Since

$$[H_D,\omega'_i]=0, \qquad (43)$$

ω'_i is a constant of motion. The eigenvalues of ω'_i and H_D are simultaneously measurable. Both $\boldsymbol{\omega}$ and H_D are four-dimensional operators. Their two-dimensional components stand for the two coupled, opposite spinning motions in a Dirac particle. While H_D stands for the total energy of the two systems as the particle energy, and $\boldsymbol{\Sigma}$ is defined by Pauli matrices preceded by the same sign, $\boldsymbol{\omega}$ stands, by its two-dimensional elements preceded by opposite signs (involved by γ^5), for some opposite quantities definitory for the two systems. So, for states of well-defined energy, the eigenvalues of ω_i to be taken into account are, unlike those of H_D, just those of its two-dimensional components. For a particle at absolute rest of Schrodinger's microscopic momentum $p_o=m_oc$, the eigenvalues of ω'_i are given by (33). They are also given by (33) for a free particle of linear momentum **p**(0,0,p), when

$$\omega'_i=2p^2c^2H_D\Sigma_i/hE^2. \qquad (44)$$

Therefore, the physical quantities associated to the two-dimensional components of ω'_i are frequencies. Their coincidence with the frequencies (33) validates the semi-classical model of Dirac particle obtained in Chs. 28, 29 as a quantum model.

Since Eq. (44) was obtained by adding the operators

$$P_\pm\omega P_\pm=\pm2p^2c^2\Sigma P_\pm/hE,$$

the only energy states $\Psi_\pm$ satisfying the eigenvalue equation of ω'_i are those also satisfying equation $h\Psi_\pm=\pm\Psi_\pm$. More generally, by Eqs. (44), the eigenvalues of ω'_i are simultaneous with those of H_D in two cases: i) for states which energy and helicity are both either positive or negative, ii) for mixed energy states and mixed helicity states. Since the Dirac eigenfunctions are linear combination of states of opposite helicities, this means that a state of 'well-defined' energy is actually an unbiased mixture of sub-states of opposite energies associated to opposite sub-spins. No evaluation of these sub-spins of the systems of subquantum particles is known at this stage of our investigation. The

main result is that the particle mass appears for the first time to be the coupling constant of these sub-spins. The particle energy appears as their coupling energy.
In accord with the commutation relations $[\omega'_i,\alpha_i]=0$ and $[S_i,\alpha_i]=0$, the eigenvalue equations of the operators $P_\pm\alpha P_\pm=\pm(cp/E)P_\pm$, associate the speeds $\pm c$ to these systems. The Zitterbewegung frequencies of the operators α_i, S_i and ω_i between states of identical p but opposite energies [57] coincide with the spinning frequencies of the model's systems. So Zitterbewegung is the rapid motion performed by peripheral subquantum particles about the systems of opposite energies, just as it is seen by an observer watching the projections of their speeds onto the coordinate axes.

34. SUBQUANTUM DETERMINATION OF DIRAC WAVEFUNCTIONS

We have shown in Sec. 32 that the Dirac wavefunctions actually contain information about the subquantum structure of the particles which they describe. To get further insight into their structure, we now relate the Dirac wavefunctions to parameters that could characterize this structure by Eqs. (31) in view of Eq. (32). Concerning a free particle moving along the third axis of coordinates, Eqs. (30) reduce to

$$i\hbar(\partial_3+\partial_o)\xi=m_oc\eta,\ i\hbar(\partial_3-\partial_o)\eta=-m_oc\xi, \qquad (45)$$

under the action of σ_3 on the spinor part of ξ and η.

The analogous Eqs. (31) and (32) enable us to describe the weakly coupled systems of subquantum particles of a Dirac particle by

$$\xi = (\rho_R)^{1/2}\exp(i\theta_R),\ \eta = (\rho_L)^{1/2}\exp(i\theta_L), \qquad (46)$$

where as functions of space and time the densities ρ_j and the phases θ_j $(j=L,R)$ determine by their variation the motion of the subquantum particles. Thus, by inserting (46) in Eqs. (45), and collecting the resulting real and imaginary parts, we get

$$\partial_o\rho_j = \varepsilon_j[\partial_3\rho_j+(2K\kappa/hc)\sin\theta],\ \partial_o\theta_j= \varepsilon_j\partial_3\theta_j-(K\kappa/hc\rho_j)\cos\theta,$$

where $\varepsilon_j=+1$ for $j=L$, $\varepsilon_j=-1$ for $j=R$, $\kappa=(\rho_L\rho_R)^{1/2}$ and $\theta=\theta_L-\theta_R$ is the relative phase. The stationary state defined by $\rho_L=\rho_R$ is governed by the equations

$$-\partial_o\rho_L = \partial_o\rho_R,\ \partial_o\theta = \partial_3(\theta_L+ \theta_R).$$

The subquantum determination of the wavefunctions by (46) was lost by their normalization.

35. PHOTON'S MODEL

The Hamiltonian [58]

$$H_{Ph}= c\cdot rot = \omega_i S_i,$$

where rot stands for rotor, $\omega_i=cp_i/\hbar$, $S_i=\hbar s_i$ and s_i are the spin matrices

$$s_1 = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{vmatrix},\ s_2 = \begin{vmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{vmatrix},\ s_3 = \begin{vmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}.,$$

is the analogous of H^o_D. The writing of H_{Ph} as a rotor, and of its wavefunctions as a superposition of wavefunctions of opposite polarizations, suggest that any photon consists of two physical entities spinning in opposite directions. The e^+-e^- annihilation suggests that these entities are also systems of subquantum particles spinning in opposite directions.

36. SUGGESTED MODEL OF SPIN-0 MESON

In view of the physical meaning of Zitterbewegung deduced in Sec. 33, the Zitterbewegung provided by the two-dimensional matrices of Sakata-Taketani equation [59] describing spin-zero mesons suggests the existence of oppositely spinning systems of subquantum particles also within these 'elementary' particles.