Essay

Unity, Oneness & Numbers: Octonion Poetry

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Abstract

The non-associative algebra of the Octonions requires an ordering and sequencing of elements similar to the syntax of language. Only certain arrangements of elements will allow the meaningful or sensible evaluation of a given expression. The translation of Octonion Maths into words results in sentences that have a profound meaning, which illustrate the grand arc of learning and the process by which knowledge is acquired. Examples of this tend to be poetic, hence the term Octonion Poetry. A discussion about the properties of the octonions and why they have stories to tell precedes a sampler of example poems that follow the same pattern.

Keywords: Octonions, non-associative, forced ordering, language, syntax, poetry.

Introduction to the Octonions

Real numbers are the most familiar, but neither the most useful number type, nor the most general. Real numbers represent a constant value, a certain amount or extent, without having to say what is represented or if it is only a fixed quantity with an abstract value. Of course, it is more common to speak of real numbers as representing real quantities – one egg or a dozen, one volt or 1.92 volts. We can have whole numbers of individual items, fractional values for part of an item, or values in between for quantities of things like water that vary continuously. But real numbers represent the constant value – the amount contained in a vessel for example – and not its freedom to vary.

Complex numbers allow us to represent both the nominal value of a quantity and the amount it can vary, as a number. Imaginary numbers were invented, or assumed to exist, as a way to calculate roots. The unit for the imaginaries is defined as $i = \sqrt{-1}$. There is no real number satisfying that expression, but if we assume that *i* is the freedom to vary by one unit in a direction orthogonal to the reals, this allows calculations that would otherwise be impossible. It also allows us to represent things like sea level – that have a median value but vary with the tides – as a single complex number, with a real part and an imaginary part. This makes complex numbers a powerful tool in Physics and for analyzing the world in general. Part of the beauty of using complex numbers this way is we retain all of the familiar tools and handy shortcuts from real-valued algebra including the associative and commutative rules.

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When it was attempted to generalize the complex numbers into higher dimensions, however, it was discovered there is a complication. Our success with complex analysis cannot be simply duplicated if we go to higher dimensions. Instead, rules of forced ordering enter the picture, because the quaternions are non-commutative. We can relate this back to modeling physical systems by saying that in addition to encoding a specific freedom to vary, each imaginary dimension encodes a direction (orthogonal to the reals), so we can think of complex numbers as representing or encoding rotation around a specific axis. When we combine rotations, things are more complicated in three dimensions or higher.

When Hamilton first discovered the quaternions, he found that three dimensions were not enough, and instead they needed to be four-dimensional with three imaginary parts. This makes the quaternions non-commutative, but it allows us to correctly represent the conditions a pilot encounters during air flight. If one were to pitch down, turn right rudder, and then bank right, one would end up back in level flight but headed in a new direction. Of course, you have to have the correct amount of pitch, yaw, and roll, for the desired result to be obtained. Those same maneuvers executed in another sequence will not keep you aloft, however, but will put your plane into a falling spiral until you hit the ground. So it is good the quaternions automatically take this into account.

It gets even more interesting going from the quaternions to the octonions, in terms of the ordering of elements or steps in a procedure. The forced ordering introduced in the quaternions takes on a new dimension in the octonions, where ordered cycles of action need to be completed in a specific sequence, for the correct result to be obtained. This is because the octonions are both non-commutative and non-associative. Of course, most of the familiar shortcuts from real-valued algebra cannot be applied, as a result. But the reason is that elements associate and commute only in specific ways, not arbitrarily.

The directionality of the octonions is stronger than that of the quaternions, though the options are also multiplied. Operations must be completed in a particular order and sequence, once a starting place is chosen, but there are many valid ways to proceed. There are 480 distinct multiplication tables by which the product of two octonion numbers is obtained [1]. But this reduces to 8 left and 8 right-hand tables once a choice is made [2], where 1 table is used for all further calculations. What the multiplication table does here is specify the ordering of elements for each subgroup, and the sequence of operations by which partial calculations are completed and combined to reach the final result. This is different from the multiplication table we are familiar with, where reversed operations yield the same value.

While this is unfamiliar territory for people working in ordinary Maths, this sort of sequentiality requirement is very familiar to those studying language. All the letters in each word must be in the correct order, and only certain arrangements of words, with a specific syntax, will make sense to someone else – even though the same words can be arranged in a variety of ways. This makes human language much more like non-commutative and non-associative Maths such as the octonions, rather than the more familiar real-valued Algebra and Arithmetic – where using the commutative and associative rules, one can arbitrarily arrange elements and simplify expressions. This is at least partly due to the fact that human language has evolved to express a sense of the conditionality in real-world processes, where antecedents are needed before a new process or process stage can be undertaken. But the grandest arena this plays out in is the acquisition of knowledge or learning itself, because there are prerequisites for anything one might seek to learn.

This is the great treasure the octonions have to offer. They show us the grand arc of all learning, and explain how knowledge is acquired, because they embody projective geometry, the geometry of perspective. If we knew how to interpret what the octonions tell us, they would reveal a higher understanding of reality and the road to higher states of consciousness. The scope of their message cuts across categories like Mathematics, Physics, Computing, Cognition, Learning, Ethics, and Religion, and appears in theories of process.

Arthur Young's book "The Reflexive Universe" [3] is subtitled "evolution of consciousness" which aptly sums up the universality of this idea. He describes the many ways form evolves through seven stages (if each stage runs to completion) across broad categories, delineating levels of abstraction and showing how the thrust of such evolution is to raise the level of consciousness from one grand cycle to the next. While he does not explicitly mention the octonions in that book, it is obvious from the details of his exposition that he speaks to their projection across categories as a driver of process evolution. I think Young was on the right track, despite shortcomings of his knowledge in some areas. He clearly shows that the appearance of a similar pattern in Physics, Chemistry, Biology, and elsewhere, is more than a coincidence. My observation is that this pattern serves to teach us how creation works and how consciousness grows as a result.

The idea that 8-dimensional algebra with 7 imaginary dimensions can teach us about learning and consciousness seems far-fetched at first. One might wonder why higher-d reality would have anything to do with consciousness in the everyday world. But it appears higher-dimensional representations occur in the brain, according to recent research [4], and that such representations are needed to sort things out in the real world – though the space we reside in is ostensibly 3-d. One researcher, Henry Markram, said "Consciousness may itself be a shadow of a higher-

dimensional structure," and he went on to explain "The mathematics usually applied to study networks cannot detect the high-dimensional structures and spaces that we now see clearly." Mystics have long described higher dimensions, and maybe there is more truth to this reality than we have believed. But in more mundane terms, higher-d Maths may aptly encode the sequentially evolutive properties we observe in so many everyday processes, and those processes may reflect an order that is inherently higher-dimensional, which appears in non-associative algebras. The idea that numbers or mathematical relations have moods has a long history, as evidenced by Leibniz writings on the trigrams of the I Ching and the binary numbers [5]. That specific ordering may also relate to the octonions. In any case, it seems likely that in other cultures, or on other worlds, Maths are taught in a way similar to how we teach writing or a new language, perhaps even as a story.

Tales of Uncle Octonius

John Baez wrote that while the reals are well-respected and dependable members of the number family and complex numbers are flashier youngsters, the quaternions are rather eccentric, and the octonions are like a crazy old uncle the rest of family doesn't talk about at all and would rather leave locked away in the attic [6]. But I like the idea of the octonions as a wealthy and eccentric uncle (who knows untold secrets) better than a weird or crazy one, any day. So in successive attempts, I honed my own story bit by bit, hoping that in the end some semblance of the truth would come through.

It immediately became apparent that an important part of the story is that the octonions are multi-faceted, so a tale of Uncle Octonius would need to weave in subtleties that arise when working with 8-d hypercomplex numbers. Octonion multiplication involves evaluating 7 ordered triplets in a specific sequence, such that any two triplets contain an element in common, and each is counted in turn. This is very much like Arthur Young's 7 committees where each committee has one member in common and they all have a say. So my first attempts to talk about Uncle Octonius and his family tried to incorporate that dynamic. It started out like this:

Whenever someone in the family talked to Uncle Octonius, they told a different story. So our family insisted that the only way anyone could be trusted to meet with him was if we went in committees of three. Good old Uncle Octonius was one crafty old fellow. Of course, he insisted that – in order to be fair – there must be a member in common with the others each time we come, so there will need to be seven committees in all. But how did he know that there would be seven committees? That crafty old genius must have planned it that way.

This seemed a bit silly, and left out instructive comments. So in my second attempt; I tried to incorporate more of John Baez' tale of the number family with the octonions as a crazy old uncle. Then the story turned out like this:

In the number family, the Reals are respected and dependable, the Complex are younger and flashier, and the Quaternions are hard to deal with because they don't commute. They say Uncle Octonius is crazy, and Cousin John says some family members want to lock him in the attic, but I think he's only so eccentric because he's seen the universe, and knows its secrets. For years we thought he wouldn't associate with the other family members at all, but eventually we worked out how to visit with him. You see, Octonius is so persuasive, he can make us do almost anything – so he can't be trusted – or at least no one person can go up to see him alone. Nobody gets the same version of the story. Even when two go at once, they always disagree on exactly what was said. Therefore, we always visit Uncle Octonius in committees of three. But, the first time a group of us visited, he insisted that he must see all the family members with equal frequency – and that there always be someone in common between any two visits. Luckily, this worked out, because there are seven of us.

The thing is, Octonius is incredibly wealthy and knows the secrets of the universe, but we were all so afraid of him we never understood why he was crazy. You see, he seemed to enjoy breaking the laws of algebra – and insisted on doing Math his own way sometimes – whenever we tried to use the associative and commutative rules to simplify expressions for him. We never knew why that was, until we tried to evaluate ourselves – thinking that both the greatest and slightest in every family group needed to be represented, within each committee, to assure trust. Octonius explained that committees have a rule that is non-commutative, where if you include everyone at once things become non-associative, because there can be disagreements between members or committees – but there is harmony with a hierarchy or ordering of individuals within each committee, if you follow the rule.

Though we're not really sure we can trust him entirely, Uncle Octonius tells us this is as fair as it can be, so he is teaching us the secrets of the universe. And who could complain? I'm glad Cousin John didn't let the others lock him up in the attic, or we would never have learned of his vast wealth and unlocked the untold secrets of the cosmos.

Introduction to the Poems

All of the octonion poems start from unity/oneness – in an undivided state – and end in a kind of reunion. There is a movement from unity to separation, then in alternating phrases of unifying

and separating expressions, coming to fruition at the end. The generating scheme is like Kant's "original unity of apperception" as unified self-consciousness, projected into the Involution and Evolution philosophy of Hegel and Sri Aurobindo [7], with a sense of things almost getting out of control and then coming back into balance. The moral or message is one of hope, where order is found again on the far shore of chaos. The pattern appears to show not only that order can arise from chaos, but that hidden order can lie buried within the chaos, and also that chaos emerges from frustration or conflicts between patterns of increasing order. This dynamic is well-explained in the book "Turbulent Mirror" [8] by Briggs and Peat. But we know from many sources that there is often a simple rule underlying the most complex behaviors. We see for example that the Mandelbrot Set, one of the most complex objects known to Math, has a formula that is exceedingly simple.

However, the pattern hidden in the octonions is in some ways simpler. While these poems came out of an attempt to map the inner worlds, and to enumerate levels of abstraction, their message is more general or universal. It was not apparent why giving a word or phrase to each level would yield congruent sentences, but once I learned that, I found I could repeat the same pattern again and again, only from a different angle. Changing the context does not change the essential meaning or reduce the profundity of the message, however. So I offer many variations on one theme, the same idea from different perspectives, and also variations of the same poem. In some ways, this piece serves as an instruction manual or template for a much larger body of work, or for writing a certain new style of poem or aphorism. If everyone knew how to create octonion poetry, I think the world would be a better place.

Octonion Poems

One
Open
As
Multiplicity and
Formless Nothingness
Finds Peace in True Relation
And Knows All as Self

Unity
Consciousness
Postulates
Numerical Order and
Chaotic disorder
Comes into Equilibrium
And is as One again

One
Conscious
Dreamer
With Many Forms, yet
Exceeding the bounds of form
Becomes Fluid and Graceful
Encompassing all that is

Unified
Accepting
Creating
Coexisting
Evolving
Resolving
Knowing

One Being
Goddess and
God
Gave Rise to Manyness
Then to Complexity beyond Reckoning
To Find the Missing Pieces of themselves
And become As One again

Identity
Awakens
Ability
Multiplies
Complexity
Finds Resolution
Becomes Self-Unity

Everything is Connected or United
Everything is Receptive or Accepting
Everything is Projective or Expressing
Everything is Compound or Multiple
Everything is Complex or Chaotic
Everything is Resolving or Reconciling
Everything is the One Being – I

Sublime Non-Difference
Great Awe and Wonder
Imagining what's Possible
Co-Creativity
Letting Go
Serene Contentment
Collectedness

Latent Attributes
Becoming Aware
Find Expression
Becoming a Great Host of Identities
and a Vast Range of Transformations
Calming a Sea of Possibilities to
Become the Awareness of All as One

Totally Ambiguous
Completely Willing
Infinitely Expressing
Multitudinally Manifest
Beyond Limitations
Thankfully Contented
As One with all my parts

Wu-Ji
Awakens into
Tai-Ji
Assuming all forms,
while moving Beyond Form to
Find Inner Peace
and Become Eternal Wisdom

Equality
Allows
Ability
Vibrancy
Flexibility
Serenity and
Kindness

A Unified Field
of Observation
Assumes a Viewpoint
Which Becomes Multiple
Then Complex beyond the Rational
Discovers the Complement of Every Quality
And Comes to Know All as One Again

Ambiguity
Observes
Creation
Multiplying
Transforming

Tranquilly gathers itself up
And is whole

Ambiguity
awakens
to find itself dreaming
assumes many viewpoints
then steps between the dimensions
and finds a way beyond all difficulties
to emerge as the One and perfect Being

Ambiguity
Opens
Projects
Becomes Compound
Then Fractal
Becomes Transparent
And Sees All as One

Love is All
We All Love
All Serve Love
All Are Many
All Evolves Complexity
All Resolves into Simplicity
All Are One

Oneness
Experience and
Power
Make Many Forms and
the Freedom to Transform where
Unfoldment settles into Patterns, becoming
the basis for Identity and the Universe of Form

Infinite Potentiality (with no possibility for perception or expression)
Gives rise to Conscious Awareness (without an object to observe)
and Creative Intelligence (without a precedent to follow)
Once sufficient order emerges (to allow measurement)
Complexity stretches the boundaries into Chaos (where order begets disorder)
Completing of the underlying theoretical framework (by finding the underlying order)
The Unique solution to all becomes apparent. (as who we really are)

Life Itself
Being Conscious of
Evenness, Self-agreement, or Self-Similarity with all beings,

wants to bestow SUFFICIENT quantities of various Qualities,
So that NOTHING may deter them
from the COMPLETE realization
of their UNIQUE Identity as the ALL-ONE

All is one $\infty = 1$
All is awakening
All is manifesting
All are many $\infty > 1$
All are complex
All is resolving
All is Self $\infty = I$

ONE BEING
GODDESS and
GOD
Gives Rise to MANYNESS
Dissolving into COMPLEXITY
Then resolving into BALANCE,
to become ONE BEING again

LOVE is our most primal nature.
We FEEL love for someone or something.
We ACT in service of our love impulses.
We are many, both as individuals and collectively, made of diverse attributes and peoples.
We are yet more than this, more than meets the eye, and more than the sum of our parts.
Love, as our true self, peacefully resolves the conflict between identities and polarities.

One who
is Open
As-If
All the Many
and what flows between
becomes the way to forge a unity
and is As One with it all

Parting Thoughts

There has been much debate over the years as to what distinguishes the creation of humans from creation by the Divine. Even in schools of thought like Advaita, where a view of the universe that is non-dualistic is aspired to, it is generally thought that Ishvara Srishti or Divine creation is somehow fundamentally different from Jiva Srishti [9], which is the creation by embodied beings. In my view, this difference is partly due to our ignorance of the natural order which prevents

humans from using the same methodology as the Divine, and not a direct consequence of our separated state. The effect of octonion poetry is to serve as a reminder that we are not separate from God, Goddess, or Spirit, and instead we have in us a blending of perfect and divine natures – regardless of how sorry our state might be. But the message of these poems and others like them is iconoclastic in a way that makes it hard for some religious folks to embrace. They bring to light the same issues raised by Dan Brown in "The Da Vinci Code" [10] where the Divine Feminine figures prominently. Many are skeptical of attributing Divine power to women, or remain resentful of attempts to question or undermine the authority of the Church. So the idea that the feminine principle lies beyond or comes before the male is sure to raise some people's ire, or will be considered heresy.

But in addition, we have a stark reminder, in these verses, that we are the Divine ourselves and that if there is evil in the world we helped create it. This is not blasphemous, but rather it is an awakening to an adult sense of responsibility for the well-being and education of others. One of my mentors as a young man, Donald Badgley, said that it was his choice in any moment to be God or the Devil, but not to abdicate or deny his power to choose. I like this metaphor as a way to face life as it is, where you don't broker away your power to do good and don't deny it is your responsibility to do so. The doctrine of radical acceptance is to see reality for what it really is, by setting aside assumptions that would limit our view or prevent us from dealing with life as it is. Reality and life are of a kind, but they are also like two sides of a coin.

For those on a spiritual journey, the inner leads to the outer and the outer leads to the inner – again and again. But the same could be said of the universe and of our mundane existence. So the lessons in octonion poetry are highly scientific. It is true we are both a multiplicity of forms (atoms and molecules) and that there is nothingness (empty space) contained in all of those forms. So remaining open, but behaving as if these things are true, is in no way unscientific. The tricky piece is the prediction that awakening to the true nature of our identity will bring us peace and unity. We shall see…

As to whether these poems have anything to do with the octonions and their algebra, I offer these thoughts. While real numbers are steady, complex numbers incorporate variation, and the quaternions and octonions increasingly display evolutive properties due to being non-commutative and non-associative. They don't just sit there unchanging, like real numbers do. This affects the objects and spaces created by these algebras. Speaking to the 'magic' of non-commutative geometry, Alain Connes said there are unexpected facts and amazing features with no counterpart in commutative spaces, explaining that "noncommutative measure spaces evolve with time" [11], and adding that they have a god-given group of automorphisms driving that

evolution. This dynamic is amplified in the octonions, since they are both non-commutative and non-associative, which P.C. Kainen said "could be a blessing in disguise.

If multiplication depends on the order of the elements being multiplied together and even on how they are grouped, then at one fell swoop, geometry enters the calculation in an organic way. The Principle of Indeterminacy could then arise in a natural fashion from relativistic considerations, making quantum theory a consequence of an underlying 8-dimensional hidden-variable process, very much in the flavor of the theories of de Broglie and Bohm. Uncertainty of measurement would be a corollary of our inability to absolutely order events or to absolutely control the way in which they are grouped." [12] In this way, the octonions might set the pattern for both how the universe unfolds and how we learn about it.

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